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Super complete-antimagicness of Amalgamation of any Graph

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Abstract*—Let H_i be a finite collection of simple, nontrivial and undirected graphs and let each H_i have a fixed vertex v_j called a terminal. The amalgamation H_i as v_j as a terminal is formed by taking all the H_i's and identifying their terminal. When H_i are all isomorphic graphs, for any positif integer $n$we denote such amalgamation by $G = Amal(H, v, n)$, where n denotes the number of copies of H. The graph G is said to be an $(a, d) - H$-antimagic total graph if there exist a bijective function $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs isomorphic to H, the total H −weights $W(H) = \sum_{v \in V(H)} f(v) + \sum_{e \in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, \dots, a + (n - 1)d\}$, where$a$ and d are positive integers and n is the number of all subgraphs isomorphic to H. An $(a, d) - H$-antimagic total labeling $f$is called super if the smallest labels appear in the vertices. In this paper, we study a super $(a, d) - H$ antimagic total labeling of $G = Amal(H, v, n)$ and its disjoint union.*

Keywords*—*Super H-antimagic total graph, Amalgamation of graph, arithmetic sequence.

I. INTRODUCTION

A graph G is said to be an $(a, d) - H$-antimagic total graph if there exist a bijective function $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, |V(G)| + |E(G)|\}$ such that for all subgraphs of Gisomorphic to H, the total H-weights $w(H) = \Sigma_{v \in V(H)} f(v) + \Sigma_{e \in E(H)} f(e)$ form an arithmetic sequence $\{a, a + d, a + 2d, \dots, a + (n - 1)d\}$, where a and $d$are positive integers and n is the number of all subgraphs of G isomorphic to$H$. If such a function exist then f is called an$(a, d) - H$-antimagic total labeling of G. An$(a, d) - H$ -antimagic total labeling f is called super if $f: V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$.

There many articles have been published in many journals, some ofthem can be cited in [2, 3, 7, 8] and [9, 10, 11, 12, 13]. For connected graph, Inayah *et a*l. in [7] proved that, for H is a non-trivial connected graph and $k \geq 2$ is an integer, $shack(H, v, k)$ which contains exactly k subgraphs isomorphic to H is H-super antimagic. They only covered a connected version of shackle of graph when a vertex as a connector, and their paper did not cover all feasible d. Our paper attempt to solve a super $(a, d) - H$antimagic total labeling of $G = Amal(H, v, n)$ and its disjoint union when H is a complete graph for feasible d. To show those existence, we will use a special technique, namely an integer set partition technique. We consider the partition $P^n_{m,d}(i, j)$ of the set $\{1, 2, \dots, mn\}$ into n columns with $n \geq 2$, m-rows such that the difference between the sum of the numbers in the $(j + 1)$th m-rows and the sum of the numbers in the jth m-rows is always equal to the constant d,where $j = 1, 2, \dots, n - 1$. The partition$P^{n,s}_{m,d}(i, j, k)$ of the set $\{1, 2, \dots, mns\}$into ns columns with $n, s \geq 2$, m-rows such that the difference between the sum of the numbers in the $(k + 1)$th m-rows and the sum of the numbers in the kth m-rows is always equal to the constant d for $j = 1, 2, \dots, n$, where $k = 1, 2, \dots, k - 1$. Thus these sums form an arithmetic sequence with the difference d. We need to establish some lemmas related to the partition $P^n_{m,d}(i, j)$and $P^{n,s}_{m,d}(i, j, k)$. These lemmas are useful to develop the super $(a, d) - H$ antimagic total labeling of $G = Amal(H, v, n)$and $G = sAmal(H, v, n)$.

II. SOME USEFUL LEMMAS

Let G be an amalgamation of any graph H, denoted by $G = Amal(H, v, n)$. The graph G is a connected graph with $|V(G)| = p_G$, $|E(G)| = q_G$, $|V(H)| = p_H$, and $E(H)| = q_H$. The vertex set and edge set of the graph $G = Amal(H, v, n)$ can be split into following sets: $V(G) = \{A\} \cup \{x_{ij};\ 1 \leq i \leq p_H - 1, 1 \leq j \leq n\}$ and $E(G) = \{e_{lj};\ 1 \leq l \leq q_H, 1 \leq j \leq n\}$. Let n, $m$be positive integers with $n \geq 2$ and $m \geq 3$. Thus $|V(G)| = p_G = n(p_H - 1) + 1$and $|E(G)| = q_G = nq_H$.

Furthermore, let G be a disjoint union of amalgamation of graph H, denoted by $G = sAmal(H, v, n)$ and s bean odd positive integer. The graph G is a disconnected graph with $|V(G)| = p_G, |E(G)| = q_G, |V(H)| = p_H$, and $|E(H)| = q_H$. The vertex set and edge set of the graph $G =$

$sAmal(H,v,n)$ can be split into following sets: $V(G) = \{A^k;\ 1 \le k \le s\} \cup \{\{x_{ij}\}^k;\ 1 \le i \le p_H - 1, 1 \le j \le n, 1 \le k \le s\}$ and $E(G) = \{\{e_{lj}\}^k;\ 1 \le j \le n, 1 \le l \le q_H, 1 \le k \le s\}$. Let n, m, and odd s be positive integerswith $n \ge 2$ and $m, s \ge 3$. Thus $|V(G)| = p_G = s(n(p_H - 1) + 1)$and $|E(G)| = q_G = snq_H$.

The upper bound of feasible d for $G = Amal(H,v,n)$ and $G = sAmal(H,v,n)$ to be a super $(a,d) - H$ −antimagic total labeling follows the following lemma [2].

Lemma 2.1 [2]

Let G be a simple graph of order p and size q. If G is super $(a,d) - H$ −antimagic total labeling then $d \le \frac{\{(p_{\{G\}} - p_{\{H\}})p_{\{H\}} + (q_{\{G\}} - q_{\{H\}})q_{\{H\}}\}}{n-1}$, for $p_{\{G\}} = |V(G)|, q_{\{G\}} = |E(G)|, p_{\{H\}} = |V(H)|, q_{\{H\}} = |E(H)|$,and $n = |H_j|$.

Corollary 2.1

For $n \ge 2$, if the graph $G = Amal(H,v,n)$ admits super $(a,d) - H$-antimagic total labeling then $d \le p_H^2 + q_H^2 - p_H$

Corollary 2.2

For $n \ge 2$ and odd $s \ge 3$, if the disconnected graph $G = sAmal(H,v,n)$ admits super $(a,d) - H$-antimagic total labeling then$d \le p_H^2 + q_H^2 - p_H + \frac{\{(s-1)p_H\}}{sn-1}$

We recall a partition $P^n_{\{m,d\}}(i,j)$ introduced in [4]. We will use the partition for a linear combination in developing a bijection of vertex and edge label of the main theorem.

Lemma 2.2[4]

Let n and $m$be positive integers. The sum of $P^n_{\{m,d\}}(i,j) = \{(i-1)n + j, 1 \le i \le m\}$ and $P^n_{\{m,d\}}(i,j) = \{(j-1)m + i; 1 \le i \le m\}$ form an aritmatic sequence of difference $d \in \{m, m^2\}$, respectively.

III. THE RESULTS

The Connected Graph

The following four lemmas are useful for the existence of super $(a,d) - H$ antimagic total labeling $G = Amal(H,v,n)$.

Lemma 3.1

Let n and m be positive integers. For $1 \le j \le n$, the sum of $P^n_{\{m,d_1\}}(i,j) = \{1 + ni - j;\ 1 \le i \le m\}$ and $P^n_{\{m,d_2\}}(i,j) = \{mn + i - mj;\ 1 \le i \le m\}$ form an arithmetic sequence of differences $d_1 = -m, d_2 = -m^2$.

Proof.

By simple calculation, for $j = 1,2,\dots,n$, it gives $\sum_{i=1}^m P^n_{\{m,d_1\}}(i,j) = P^n_{\{m,d_1\}}(j) \leftrightarrow P^n_{\{m,d_1\}}(j) = \{\frac{n}{2}(m^2 + m) + m - mj\}\} \leftrightarrow P^n_{\{m,d_1\}}^{n}(j) = \{\frac{n}{2}(m^2 + m), \frac{n}{2}(m^2 + m) - m, \frac{n}{2}(m^2 + m) - 2m, \dots, \frac{n}{2}(m^2 + m)m - mn\}$ and $\sum_{i=1}^m P^n_{\{m,d_2\}}(j) \leftrightarrow P^n_{\{m,d_2\}}(j) \leftrightarrow P^n_{\{m,d_2\}}(j) = \{\{\frac{m}{2}(2mn + m + 1)\} - m^2 j\} \leftrightarrow P^n_{\{m,d_2\}}(j) = \{\frac{m}{2}(2mn + m + 1) - m^2, \frac{m}{2}(2mn + m + 1) - 2m^2, \dots, \frac{m}{2}(2mn + m + 1) - m^2 n\}$. Itis easy to see that the differences of those sequences are $d_1 = -m, d_2 = -m^2$. It concludes the proof. ■

Lemma 3.2

Let n and m be positive integers. For $1 \le j \le n$, the sum of

$$P^n_{\{m,d_3\}}(i,j) = \begin{cases} i + (j-1)m;\ 1 \le i \le m; i\ odd \\ n(i-2) + 2j;\ 1 \le i \le m; i\ even \end{cases}$$

form an aritmatic sequence of difference $d_3 = \frac{1}{2}m^2 + m$.

Proof.

By simple calculation, it gives $\sum_{i=1}^m P^n_{\{m,d_3\}}(i,j) = P^n_{\{m,d_3\}}(j) \leftrightarrow P^n_{\{m,d_3\}}(j) = \left\{\left(\frac{m^2}{2} + m\right)j + \frac{m}{4}(mn - 2n - m)\right\} \leftrightarrow P^n_{\{m,d_3\}}(j) = \{(\frac{m^2}{2} + m) + \frac{m}{4}(mn - 2n - m), (m^2 + 2m) + \frac{m}{4}(mn - 2n - m), \dots, (\frac{m^2}{2} + m)n + \frac{m}{4}(mn - 2n - m)\}$. It concludes the proof. ■

Lemma 3.3

Let n and m be positive integers. For $1 \le j \le n$, the sum of

$$P^n_{\{m,d_4\}}(i,j) = \begin{cases} mn - mj + i;\ 1 \le i \le m; i\ odd \\ ni + 2 - 2j;\ 1 \le i \le m; i\ even \end{cases}$$

form an aritmatic sequence of difference $d_4 = -\left(\frac{1}{2}m^2 + m\right)$.

Proof.

By simple calculation, it gives $\sum_{i=1}^m P^n_{\{m,d_4\}}(i,j) = P^n_{\{m,d_4\}}(j) \leftrightarrow P^n_{\{m,d_4\}}(j) = \left\{\frac{m}{4}(3mn + m + 2n + 4) - \left(\frac{m^2}{2} + m\right)\right\} \leftrightarrow P^n_{\{m,d_4\}}(j) = \left\{\frac{m}{4}(3mn + m + 2n + 4) - \frac{m^2}{2} - m\right), \left(\frac{m}{4}(3mn + m + 2n + 4) - m^2 - 2m\right), \dots, \frac{m}{4}(3mn + m + 2n + 4) - \frac{3m^2}{2} - 3m)\}$.We have the desired difference. ■

Now we are ready to present the main theorem related to the existence of super $(a,d) - H$ antimagicness of the connected graph $G = Amal(H,v,n)$, in the following theorem.

Theorem 3.1

For $n \ge 2$, the graph $G = Amal(H,v,n)$ admits a super $(a,d) - H$ antimagic total labeling with feasible $d = m_1 - m_2 + m_3^2 - m_4^2 + \frac{1}{2}m_5^2 + m_5 - \left(\frac{1}{2}m_6^2 + m_6\right) + r_1 - r_2 + r_3^2 - r_4^2 + \left(\frac{1}{2}r_5^2 + r_5\right) - \left(\frac{1}{2}r_6^2 + r_6\right)$

Proof.

Let m and r be positive integers, with $m = p_H - 1$and $r = q_H$. For $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$, by Lemma 2.2, 3.1, 3.2 and 3.3 we define the vertex and the edge labels as a linear combination of $P^n_{\{m_1,m_1\}}(i,j)$; $P^n_{\{m_2,-m_2\}}(i,j)$;$P^n_{\{m_2,-m_2\}}(i,j)$;$P^n_{\{m_3,m_3^2\}}(i,j)$;

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$P^n_{\{m_4,-m_4^2\}}(i,j)$; $P^n_{\{m_5,\frac{1}{2}m_5^2+m_5\}}(i,j)$;

and $P^n_{\{m_6,-\left(\frac{1}{2}m_6^2+m_6\right)\}}(i,j)$ as follows:

$f\_1(A) = 1$, and

$$f_1\,(x_{\{i,j\}}) = \{P^n_{\{m_1,m_1\}} \oplus 1\} \cup \{P^n_{\{m_2,-m_2\}} \oplus [n(m_1) + 1]\}$$
$$\cup \left\{P^n_{\{m_3,m_3^2\}} \oplus [n(m_1 + m_2) + 1]\right\}$$
$$\cup \left\{P^n_{\{m_4,-m_4^2\}} \oplus [n(m_1 + m_2 + m_3) + 1]\right\}$$
$$\cup \left\{P^n_{\{m_5,\frac{1}{2}m_5^2+m_5\}} \oplus [n\,\Sigma^4_{\{t=1\}}m_t + 1]\right\}$$
$$\cup\ \{P^n_{\{m_6,-\left(\frac{1}{2}m_6^2+m_6\right)\}} \oplus [n\,\Sigma^5_{\{t=1\}}m_t + 1]\}$$

$$f_1(e_{\{l,j\}}) = \{P^n_{\{r_1,r_1\}} \oplus [mn + 1]\}$$
$$\cup \{P^n_{\{r_2,-r_2\}} \oplus [n(r_1) + mn + 1]\}$$
$$\cup \left\{P^n_{\{r_3,r_3^2\}} \oplus [n(r_1 + r_2) + mn + 1]\right\}$$
$$\cup \left\{P^n_{\{r_4,-r_4^2\}} \oplus [n(r_1 + r_2 + r_3) + mn + 1]\right\}$$
$$\cup \left\{P^n_{\{r_5,\frac{1}{2}r_5^2+r_5\}} \oplus [n\,\Sigma^4_{\{t=1\}}r_t + mn + 1]\right\}$$
$$\cup\ \{P^n_{\{r_6,-\left(\frac{1}{2}r_6^2+r_6\right)\}} \oplus [n\,\Sigma^5_{\{t=1\}}r_t + mn + 1]\}$$

The vertex labeling f is a bijective function$f: V(G) \cup E(G) \rightarrow \{1,2,\ldots,p_G + q_G\}$. The total edge-weights of$G = Amal(H,v,n)$ under the labeling f, for $j = 1,2,\ldots,n$, constitute the following sets:

$$W_{\{f_1\}} = \sum f_1(A) + \sum f_1\,(x_{\{i,j\}}) + \sum f_1(e_{\{l,j\}})$$

$$= C^n_{\{m,d\}} + [m_1 - m_2 + m_3^2 - m_4^2 + \frac{1}{2}m_5^2 + m_5 - \left(\frac{1}{2}m_6^2 + m_6\right) + r_1 - r_2 + r_3^2 - r_4^2 + \left(\frac{1}{2}r_5^2 + r_5\right)$$

It is easy that the set of total edge-weights $W_{\{f_1\}}$ consists of an arithmetic sequence of the smallest value a and the difference $d = m_1 - m_2 + m_3^2 - m_4^2 + \frac{1}{2}m_5^2 + m_5 - \left(\frac{1}{2}m_6^2 + m_6\right) + r_1 - r_2 + r_3^2 - r_4^2 + \left(\frac{1}{2}r_5^2 + r_5\right) - \left(\frac{1}{2}r_6^2 + r_6\right)$.Since the biggest d is attained when $d = m^2 + r^2$then, form $= p_H - 1$and $r = q_H$, it gives $0 \leq d \leq p_H^2 + q_H^2 - p_H \leftrightarrow 0 \leq (p_H - 1)^2 + q_H^2 \leq p_H^2 + q_H^2 - p_H \leftrightarrow 0 \leq p_H^2 + q_H^2 - p_H - (p_H - 1) \leq p_H^2 + q_H^2 - p_H$. It concludes the proof. ■

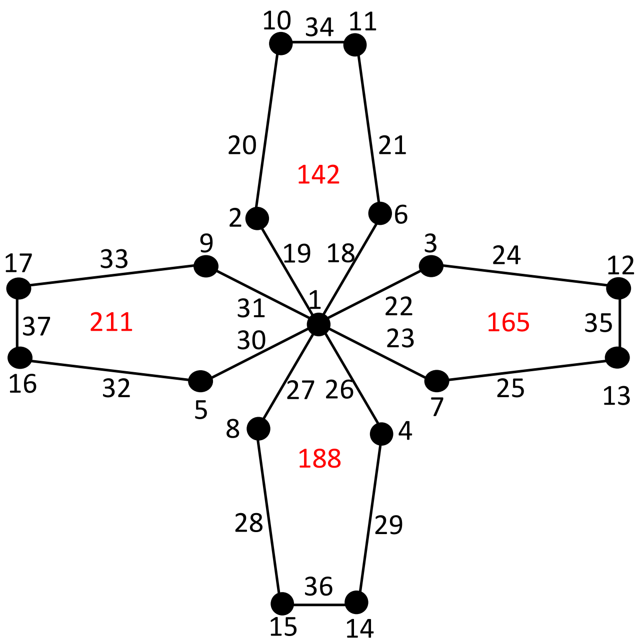

Fig.1: Super (142,23)-H- antimagic total covering of graph G=Amal(C5, v, 4)

The Disjoint Union Graph

The following lemmas are useful for the existence of super $(a,d) - H$ antimagic total labeling disjoint union of the graph G, denoted by $G = sAmal(H,v,n)$.

Lemma 3.5

Let n, m and s be positive integers $1 \leq j \leq n$; $1 \leq k \leq s$, the sum of $P^{\{n,s\}}_{\{m,d_7\}}(i,j,k) = \{(k-1)m + i + (j-1)ms;\ 1 \leq i \leq m;\ 1 \leq j \leq n;\ 3 \leq k \leq s\}$ and the sum of $P^{\{n,s\}}_{\{m,d_8\}}(i,j,k) = \{(j-1)s + i + k + (i-1)ns;\ 1 \leq i \leq m\}$ form an arithmetic sequence of differences $d_5 = m^2$and $d_6 = m$.

Proof.

By simple calculation, for $j = 1,2,\ldots,n$, it gives $\Sigma^m_{i=1}P^{\{ns\}}_{\{m,d_5\}}(j,k) = P^{\{ns\}}_{\{m,d_5\}}(k) \leftrightarrow P^{\{ns\}}_{\{m,d_5\}}(j,k) = \left\{\frac{1}{2}(m - m^2) + m^2k + m^2s(j-1)\right\} \leftrightarrow P^{\{ns\}}_{\{m,d_5\}}(j,k) = \{\frac{1}{2}(m - m^2) + m^2, \frac{1}{2}(m - m^2) + 2m^2, \ldots, \frac{1}{2}(m - m^2) + sm^2, \frac{1}{2}(m - m^2) + (s+1)m^2, \frac{1}{2}(m - m^2) + (s+2)m^2, \ldots, \frac{1}{2}(m - m^2) + snm^2\}$. Furthermore $\Sigma^m_{i=1}P^{\{n,s\}}_{\{m,d_6\}}(j,k) = P^{\{n,s\}}_{\{m,d_6\}}(k) \leftrightarrow P^{\{n,s\}}_{\{m,d_6\}}(j,k) = \left\{\frac{ns}{2}(m^2 - m) + m((j-1)s + k)\right\} \leftrightarrow P^{\{n,s\}}_{\{m,d_6\}}(j,k) = \{\frac{ns}{2}(m^2 - m) + m, \frac{ns}{2}(m^2 - m) + 2m, \ldots, \frac{ns}{2}(m^2 - m) + sm, \frac{ns}{2}(m^2 - m) + (s+1)m, \frac{ns}{2}(m^2 - m) + (s+2)m, \ldots, \frac{ns}{2}(m^2 - m) + snm\}$. It concludes the proof.

■

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Lemma 3.6

Let ,m and $s$be positive integers $1 \leq j \leq n$; $1 \leq k \leq s$, the sum of $P^{\{n,s\}}_{\{m,d_7\}}(i,j,k) = \{ms + i - mk + (n - j)ms;\ 1 \leq i \leq m\}$ and the sum of$P^{\{n,s\}}_{\{m,d_8\}}(i,j,k) = \{i - k - s(j-1) + (ns)i;\ 1 \leq i \leq m;\ 1 \leq j \leq n;\ 3 \leq k \leq s\}$ form an arithmetic sequences of difference $d_7 = -m^2$and $d_8 = -m$.

Proof.

By simple calculation, for $j = 1,2,\dots,n$, it gives $\Sigma_{i=1}^{m} P^{\{n,s\}}_{\{m,d_7\}}(j,k) = P^{\{n,s\}}_{\{m,d_7\}}(k) \leftrightarrow P^{\{n,s\}}_{\{m,d_7\}}(j,k) = \left\{\frac{m}{2}(2ms + m + 1) + m^2sn - m^2(js + k)\right\} \leftrightarrow P^{\{n,s\}}_{\{m,d_7\}}(j,k) = \{\frac{m}{2}(2ms + m + 1) + m^2sn - m^2(s + 1), \frac{m}{2}(2ms + m + 1) + m^2sn - m^2(s + 2), \dots, \frac{m}{2}(2ms + m + 1) + m^2sn - m^2(2s), \frac{m}{2}(2ms + m + 1) + m^2sn - m^2(2s + 1), \dots, \frac{m}{2}(2ms + m + 1) + m^2sn - m^2(ns + s)\}$. Furthermore $\Sigma_{i=1}^{m} P^{\{n,s\}}_{\{m,d_8\}}(j,k) = P^{\{n,s\}}_{\{m,d_8\}}(k) \leftrightarrow P^{\{n,s\}}_{\{m,d_8\}}(j,k) = \left\{\frac{1}{2}(m^2 + m)(ns + 1) + ms - m(js + k)\right\} \leftrightarrow P^{\{n,s\}}_{\{m,d_8\}}(j,k) = \{\frac{1}{2}(m^2 + m)(ns + 1) + ms - m(s + 1), \frac{1}{2}(m^2 + m)(ns + 1) + ms - m(s + 2), \dots, \frac{1}{2}(m^2 + m)(ns + 1) + ms - m(2s), \frac{1}{2}(m^2 + m)(ns + 1) + ms - m(2s + 1), \dots, \frac{1}{2}(m^2 + m)(ns + 1) + ms - m(ns + s)\}$. It concludes the proof. ■

Now we are ready to present the main theorem related to the existence of super $(a,d) - H$ antimagicness of the disconnected graph$G = sAmal(H,v,n)$, in the following theorem.

Theorem 3.2

For $n \geq 2$ and odd $s \geq 3$, the graph $G = sAmal(H,v,n)$ admits a super $(a,d) - H$ antimagic total labeling with feasible $d = m_1 - m_2 + m_3^2 - m_4^2 + r_1 - r_2 + r_3^2 - r_4^2$.

Proof.

For $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$,by Lemma 3.5 and 3.6 we define the vertex and the edge labels as a linear combination of $P^{\{n,s\}}_{\{m_1,m_1\}}(i,j,k), P^{\{n,s\}}_{\{m_2,-m_2\}}(i,j,k), P^{\{n,s\}}_{\{m_3,m_3^2\}}(i,j,k)$, and $P^{\{n,s\}}_{\{m_4,-m_4^2\}}(i,j,k)$as follows:

$f_1(A^k) = k$, and for $i = 1,2,\dots,m-2$

$$f_2(x^k_{\{i,j\}}) = \left\{P^{\{n,s\}}_{\{m_1,m_1\}} \oplus s\right\} \cup \left\{P^{\{n,s\}}_{\{m_2,-m_2\}} \oplus s(nm_1 + 1)\right\} \cup \left\{P^{\{n,s\}}_{\{m_3,m_3^2\}} \oplus s(n(m_1 + m_2) + 1)\right\} \cup \{P^{\{n,s\}}_{\{m_4,-m_4^2\}} \oplus s(n(m\_1 + m\_2 + m\_3) + 1)\}$$

$$f_2(x^k_{\{i,j\}}) = \begin{cases} s(n+1) + 1 - js - 2k + s[n(m-2) + 1], \\ \quad for\ 1 \leq k \leq \frac{\{s-1\}}{2}, i = m - 1 \\ s(n+2) + 1 - js - 2k + s[n(m-2) + 1], \\ \quad for\ \frac{s+1}{2} \leq i \leq s, i = m - 1 \\ \frac{1}{2}(1 - s) + js + k + ns + s[n(m-2) + 1], \\ \quad for\ \frac{s+1}{2} \leq k \leq s, i = m \\ \frac{1}{2}(1 - 3s) + js + k + ns + s[n(m-2) + 1], \\ \quad for\ \frac{s+1}{2} \leq k \leq s, i = m \end{cases}$$

$$f_2(e^k_{\{l,j\}}) = \left\{P^{\{n,s\}}_{\{r_1,r_1\}} \oplus s[(mn + 1)]\right\} \cup \left\{P^{\{n,s\}}_{\{r_2,-r_2\}} \oplus s[(n(r_1) + mn + 1)]\right\} \cup \{P^{\{n,s\}}_{\{r_3,r_3^2\}} \oplus s[(n(r\_1)) + n(r_2) + mn + 1)]\} \cup \{P^{n,s}_{\{r_4 - r_4{}^2\}} \oplus s[(n(r_1) + n(r_2) + n(r_3) + mn + 1)]\}$$

The vertex labeling f_2 is a bijective function $f_2: V(G) \cup E(G) \to \{1,2,\dots,p_G + q_G\}$. The total edge-weights of $G = sAmal(H,v,n)$ under the labeling f, for $j = 1,2,\dots,n$ and $k = 1,2,\dots,s$, constitute the following sets:

$$W_{\{f_2\}} = \sum f_2(A^k) + \sum f_2(x^k_{\{i,j\}})_{1 \leq i \leq m-2} + \sum f_2(x^k_{\{i,j\}})_{m-1 \leq i \leq m} + \sum f_1(e^k_{\{l,j\}})$$

$$W_{\{f_2\}} = \{C^{\{n,s\}}_{\{m,d\}} + [m_1 - m_2 + m_3^2 - m_4^2 + r_1 - r_2 + r_3^2 - r_4^2]\}(js + k)$$

It is easy that the set of total edge-weights $W_{\{f_2\}}$ consists of an arithmetic sequence of the smallest value a and the difference $d = m_1 - m_2 + m_3^2 - m_4^2 + r_1 - r_2 + r_3^2 - r_4^2$. Since the biggest d is attained when $d = m^2 + r^2$ then, for $m = p_H - 3$ and $r = q_H$, it gives $0 \leq d \leq p_H^2 + q_H^2 - p_H + \frac{(s-1)p_H}{sn-1} \leftrightarrow 0 \leq (p_H - 3)^2 + q_H^2 \leq p_H^2 + q_H^2 - p_H + \frac{(s-1)p_H}{sn-1}$ It concludes the proof. ■

IV. CONCLUSION

We have shown the existence of super antimagicness of amalgamation of complete graph H, denoted by $G = Amal(H,v,n)$ for connected one and $G = sAmal(H,v,n)$ for disconnected one, where H is any graph. By using a partition technique we can prove that $Amal(H,v,n)$ admits a super $(a,d) - H$ antimagic total labeling with $d = m_1 - m_2 + m_3^2 - m_4^2 + \frac{1}{2}m_5^2 + m_5 - \left(\frac{1}{2}m_6^2 + m_6\right) + r_1 - r_2 + r_3^2 - r_4^2 + \left(\frac{1}{2}r_5^2 + r_5\right) - \left(\frac{1}{2}r_6^2 + r_6\right)$, but for $G = sAmal(H,v,n)$, the existence of its super antimagicness only holds for s odd with $d = m_1 - m_2 + m_3^2 - m_4^2 + r_1 - r_2 + r_3^2 - r_4^2$. We also note that if the amalgamation of complete graph H contains a subgraph as a connector then finding the labels for feasible d remains

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widely open. Thus, we propose the following open problem.

Open Problem

Let H be a subgraph of $G$and $G = sAmal(H, v, n)$. For s even, does G admit a super $(a, d) - H$ antimagic total labeling for$n \geq$ 2 and feasible d?

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